Hindawi
Advances in Mathematical Physics
Volume 2018, Article ID 2031942, 10 pages
https://doi.org/10.1155/2018/2031942

Research Article

Combination-Combination Projective Synchronization of Multiple Chaotic Systems Using Sliding Mode Control

Junwei Sun ,[1,2] Nan Li,[1,2] and Jie Fang [1,2]

[1] College of Electronic and Information Engineering, Zhengzhou University of Light Industry, Zhengzhou 450002, China
[2] Henan Key Lab of Information-Based Electrical Appliances, Zhengzhou University of Light Industry, Zhengzhou 450002, China

Correspondence should be addressed to Jie Fang; fang0511jie@126.com

Received 5 February 2018; Revised 12 July 2018; Accepted 18 July 2018; Published 1 August 2018

Academic Editor: Xavier Leoncini

Based on projective synchronization and combination synchronization model, a type of combination-combination projective synchronization is realized via nonsingular sliding mode control technique for multiple different chaotic systems. Concretely, on the basic of the adaptive laws and stability theory, the corresponding sliding mode control surfaces and controllers are designed to achieve the combination-combination projective synchronization between the combination of two chaotic systems as drive system and the combination of multiple chaotic systems as response system with disturbances. Some criteria and corollaries are derived for combination-combination projective synchronization of the multiple different chaotic systems. Finally, the numerical simulation results are presented to demonstrate the effectiveness and correctness of the synchronization scheme.

1. Introduction

With the development of the control area, chaotic synchronization has become an important and significance research direction in nonlinear science due to its potential practical application in many scientific and engineering fields such as electronic circuits [1], biological systems [2], and communication systems [3] during the recent years. From the viewpoint of control, synchronization of chaotic systems [4] is a hard task because of their nonlinear behavior and sensitivity to the initial values. It is due to the fact that for chaotic systems, with complex mathematical expression, stability analysis methods are difficult to be implemented. Synchronization of chaotic systems refers to a process wherein two or many chaotic systems adjust a given synchronization property of their motion to a common behavior due to a coupling or to a forcing. The main idea of chaos synchronization is to make the states of the response system follow the states of the drive system asymptotically.

Up to now, various kinds of synchronization have been intensively investigated and a lot of theoretical results have been obtained, such as complete synchronization [5, 6], anti-synchronization [7], phase synchronization [8], generalized synchronization [9–12], antiphase synchronization [13], lag synchronization [14], partial synchronization [15], projective synchronization [16–19], time scale synchronization [20], combination synchronization [21–23], and compound synchronization [24]. Projective synchronization has attracted more and more attention due to the flexibility and adjustability of the proportionality factors, where the complexity of the system is enhanced and increases the secrecy property of the system. In this period, some projective synchronization types have been developed. Modified projective synchronization has been studied for a class of real nonlinear systems and a class of complex nonlinear systems [25–27]. On another research frontier, the combination synchronization has been investigated for secure communication, which can double the number of variables of the chaotic systems to strengthen the contents and security of the transmitted information. For instance, a method has been designed to realize combination synchronization of three different chaotic complex systems, where the combination system of two drive systems can synchronize with one response system [28–30]. And, a novel kind of compound synchronization among four chaotic systems is investigated to enhance the security of communication, where the compound system of three drive systems can realize the synchronization with one response system [31, 32]. Although the combination synchronization of chaotic systems has more advantages and potential compared to the synchronization between one drive system and one response system, the study of combination synchronization has been mostly concerned with the synchronization of one response system and does not consider the influences of some uncertainties parameters for multiple response systems.

On the other hand, several control methods have been investigated to realize chaos synchronization such as feedback control method [33], active control method [34], back stepping method [35], adaptive control method [36–38], impulsive control method [39], coupling control method [40], and sliding mode control method [41]. The concept and application of sliding mode control have become a popular research subject among many control theories. The difference between sliding mode control [42, 43] and conventional control lies in the discontinuity of control. It is a nonlinear control, which is realized by switching functions. The structure of the controller is switched according to the degree of system state deviating from the sliding mode, so that the system runs according to the rule of sliding mode. By designing a switching surface and applying a discontinuous control law, the trajectories of dynamic systems can be forced to slide along the desired sliding surface. The advantage is that the controlled system with sliding mode control exhibits stability properties with respect to both internal parameter uncertainties and external disturbances, which has been applied to cope with the control problem of chaotic system [44]. To the best of our knowledge, so far, the studies of chaos synchronization have little involvement with the synchronization scheme that combines projection synchronization and combination synchronization. The research on the combination-combination projective synchronization of multiple drive and multiple response chaotic systems with unknown disturbances is still an opportunity and challenge in the field of nonlinear science.

Motivated by the above discussions, a novel kind of combination-combination projective synchronization among multiple chaotic systems is investigated via the nonsingular sliding mode control, where the mixed system of two drive systems can realize the synchronization with multiple response systems. Moreover, the adaptive laws are proposed and sliding mode controllers are designed for the synchronization of uncertain chaotic systems with unknown parameters and external disturbances. Numerical simulation results are shown to verify the effectiveness and applicability of the combination-combination projective synchronization.

Compared with prior works [5, 6, 16], there are two advantages which make our paper more attractive and meaningful. At firstly, the paper extends projective synchronization of chaotic systems to combination-combination projective synchronization of chaotic systems, which make the dynamic behaviors and variables of the system more abundant and can greatly improve the security of communication. And importantly, the synchronization among multiple chaotic systems with unknown disturbances is discussed and realized by designing corresponding sliding mode surfaces and controllers, which have a better control performance than some existing results [5, 6, 16]. Hence, the results of this paper may extend and improve existing results in some extent.

The organization of this work is organized as follows. Section 2 shows a scheme of modified combination-combination projective synchronization. In Section 3, sliding mode surface and controllers are designed and stability is analysed. In Section 4, the simulation example is realized to validate the effectiveness and the feasibility of the proposed control strategy. Finally, the concluding remarks are given in Section 5.

2. Combination-Combination Projective Synchronization of Multiple Chaotic Systems with Disturbances

2.1. Problem Formulation. In this section, the combination-combination projective synchronization problem of multiple chaotic systems is proposed, which has two drive systems that synchronize multiple response systems. The precise definitions of combination-combination projective synchronization scheme and some definitions are introduced.

Without loss of generality, consider the following N chaotic systems with disturbances; the first system is drive system and is described by

$$\dot{x}_1(t) = A_1 x_1(t) + f_1(x_1(t)) + D_1(t), \tag{1}$$

where $x_1(t) = [x_{11}(t), x_{12}(t), \ldots, x_{1n}(t)]^T$ is the state vector of the first chaotic drive system (1). $f_1(x_1(t))$ is the continuous nonlinear function, $f_1(x_1(t)) = [f_{11}(x_1(t)), f_{12}(x_1(t)), \ldots, f_{1n}(x_1(t))]^T$, $A_1 = [A_{11}, A_{12}, \ldots, A_{1n}]^T$ is a coefficient matrix, and $A_{11}, A_{12}, \ldots, A_{1n}$ are an $n \times 1$ real vector of system parameters, respectively. The disturbance of the chaotic system (1) is defined as $D_1(t) = [d_{11}(t), d_{12}(t), \cdots d_{1n}(t)]^T$, which is an $n\times1$ vector.

The corresponding $N-1$ systems for system (1) with control inputs are response systems and can be written as follows:

$$\begin{aligned} \dot{x}_2(t) &= A_2 x_2(t) + f_2(x_2(t)) + D_2(t) + u_1(t), \\ &\vdots \\ \dot{x}_N(t) &= A_N x_N(t) + f_N(x_N(t)) + D_N(t) + u_{N-1}(t), \end{aligned} \tag{2}$$

where $x_j(t) = [x_{j1}(t), x_{j2}(t), \ldots, x_{jn}(t)]^T$ is the state vector of system (2). $f_j(x_j(t)) = [f_{j1}(x_j(t)), f_{j2}(x_j(t)), \ldots, f_{jn}(x_j(t))]^T$, $f_{j1}(x_j(t)), f_{j2}(x_j(t)), \ldots, f_{jn}(x_j(t))$ are the continuous nonlinear functions.

$A_j = [A_{j1}, A_{j2}, \ldots, A_{jn}]^T$ is a coefficient matrix, $A_{j1}, A_{j2}, \ldots, A_{jn}$ are an $n\times1$ real vector of system parameters, respectively. $D_j(t) = [d_{j1}(t), d_{j2}(t), \ldots, d_{jn}(t)]^T$ is the disturbance of the system (2), which is an $n\times1$ vector. The control input is $u_{j-1}(t) = [u_{j-1,1}(t), u_{j-1,2}(t), \cdots, u_{j-1,n}(t)]^T$, $(j = 2, 3, 4 \cdots, N)$.

The second drive system is given

$$\dot{y}_1(t) = B_1 y_1(t) + g_1(y_1(t)) + K_1(t), \tag{3}$$

where $y_1(t) = [x_{11}(t), y_{12}(t), \ldots, y_{1n}(t)]^T$ is the state vector of the chaotic drive system (3). $g_1(y_1(t))$ is the continuous nonlinear function, $g_1(y_1(t)) = [g_{11}(y_1(t)), g_{12}(y_1(t)), \ldots, g_{1n}(y_1(t))]^T$. $B_1 = [B_{11}, B_{12}, \ldots, B_{1n}]^T$ is a coefficient matrix; $B_{11}, B_{12}, \ldots, B_{1n}$ are an $n\times 1$ real vector of system parameters, respectively. The disturbance of the chaotic system (3) is defined as $K_1(t) = [k_{11}(t), k_{12}(t), \cdots k_{1n}(t)]^T$, which is an $n\times 1$ vector.

The corresponding $N-1$ response systems with control inputs can be written as follows:

$$\begin{aligned} \dot{y}_2(t) &= B_2 y_2(t) + g_2(y_2(t)) + K_2(t) + w_1(t), \\ &\vdots \\ \dot{y}_N(t) &= B_N y_N(t) + g_N(y_N(t)) + K_N(t) + w_{N-1}(t), \end{aligned} \tag{4}$$

where $y_j(t) = [y_{j1}(t), y_{j2}(t), \ldots, y_{jn}(t)]^T$ is the state vector of the system (4). $g_j(y_j(t)) = [y_{j1}(y_j(t)), g_{j2}(y_j(t)), \ldots, g_{jn}(y_j(t))]^T$. $g_{j1}(y_j(t)), g_{j2}(y_j(t)), \ldots, g_{jn}(y_j(t))$ are the continuous nonlinear functions.

$B_j = [B_{j1}, B_{j2}, \ldots, B_{jn}]^T$ is a coefficient matrix; $B_{j1}, B_{j2}, \ldots, B_{jn}$ are an $n\times 1$ real vector of system parameters, respectively. $K_j(t) = [k_{j1}(t), k_{j2}(t), \ldots, k_{jn}(t)]^T$ is the disturbance of the system (4), which is an $n\times 1$ vector. The control input is $w_{j-1}(t) = [w_{j-1,1}(t), w_{j-1,2}(t), \cdots, w_{j-1,n}(t)]^T$, $(j = 2, 3, 4\cdots, N)$.

2.2. The Theory of Combination-Combination Projective Synchronization. In the section, we firstly design the scheme of combination-combination projective synchronization in our drive-response synchronization scheme with two drive systems and multiple response system. It is assumed that the system $x_1(t)$ is the drive system, and the other $N-1$ systems $x_2(t), \ldots, x_N(t)$ are response systems. The system $y_1(t)$ is the drive system, and the other $N-1$ systems $y_2(t), \ldots, y_N(t)$ are response systems. Therefore, the following definition 1 will be given.

Definition 1. Consider the combination of systems $x_1(t)$ and $y_1(t)$, and the combination of response systems $x_2(t)$ and $y_2(t)$, the combination of response systems $x_3(t)$ and $y_3(t), \cdots$, the combination of response systems $x_j(t)$ and $y_j(t)$ with unknown disturbances, respectively. If the time t goes to infinity, such that

$$\begin{aligned} \lim_{t\longrightarrow\infty} \left\|e_{j-1}(t)\right\| &= \left\|x_1(t) + y_1(t) - J_j\left[x_j(t) + y_j(t)\right]\right\| \\ &= 0, \end{aligned} \tag{5}$$

the synchronization is called combination-combination projective synchronization, and $\|\cdot\|$ represents the matrix vector norm. $e_{j-1}(t)$ is combination-combination projective synchronization error and $e_{j-1}(t) = [e_{j-1,1}(t), e_{j-1,2}(t), \ldots, e_{j-1,n}(t)]^T$, $J_j = \mathrm{diag}[\xi_{j1}, \xi_{j2}, \ldots, \xi_{jn}]$ is a diagonal matrix and $\xi_{j1}, \xi_{j2}, \ldots, \xi_{jn}$ are the scaling factors $(j = 2, 3, \ldots, N)$.

The dynamics system errors can be further obtained as follows:

$$\begin{aligned} \dot{e}_{j-1,1}(t) &= A_{11}x_1(t) + f_{11}(x_1(t)) + d_{11}(t) + B_{11}y_1(t) \\ &\quad + g_{11}(y_1(t)) + k_{11}(t) - \xi_{j1}\left[A_{j1}x_j(t)\right. \\ &\quad + f_{j1}\left(x_j(t)\right) + d_{j1}(t) + u_{j-1,1}(t) + B_{j1}y_j(t) \\ &\quad \left.+ g_{j1}\left(y_j(t)\right) + k_{j1}(t) + w_{j-1,1}(t)\right], \\ \dot{e}_{j-1,2}(t) &= A_{12}x_1(t) + f_{12}(x_1(t)) + d_{12}(t) + B_{12}y_1(t) \\ &\quad + g_{12}(y_1(t)) + k_{12}(t) - \xi_{j2}\left[A_{j2}x_j(t)\right. \\ &\quad + f_{j2}\left(x_j(t)\right) + d_{j2}(t) + u_{j-1,2}(t) + B_{j2}y_j(t) \\ &\quad \left.+ g_{j2}\left(y_j(t)\right) + k_{j2}(t) + w_{j-1,2}(t)\right], \\ &\vdots \\ \dot{e}_{j-1,n}(t) &= A_{1n}x_1(t) + f_{1n}(x_1(t)) + d_{1n}(t) \\ &\quad + B_{1n}y_1(t) + g_{1n}(y_1(t)) + k_{1n}(t) - \xi_{jn}\left[A_{jn}x_j(t)\right. \\ &\quad + f_{jn}\left(x_j(t)\right) + d_{jn}(t) + u_{j-1,n}(t) + B_{jn}y_j(t) \\ &\quad \left.+ g_{jn}\left(y_j(t)\right) + k_{jn}(t) + w_{j-1,n}(t)\right]. \end{aligned} \tag{6}$$

In order to realize the combination-combination projective synchronization, it needs to design controllers $u_1(t), u_2(t), \ldots, u_{j-1}(t)$ and $w_1(t), w_2(t), \ldots, w_{j-1}(t)$ make all the trajectories $x_1(t), x_2(t), \ldots, x_j(t)$ and $y_1(t), y_2(t), \ldots, y_j(t)$ in any initial conditions $x_1(0), x_2(0), \ldots, x_j(0)$ and $y_1(0), y_2(0), \ldots, y_j(0)$ satisfy the following condition: $\lim_{t\longrightarrow\infty}\|e_{j-1}\| = \|x_1(t) + y_1(t) - J_jx_j(t) - J_jy_j(t)\| = 0$, $(j = 2, 3, 4\cdots, N)$.

Assumption 2. The external disturbances are norm-bounded, that is, $\|D_1(t)\| \le \alpha_1$, $\|J_jD_j(t)\| \le \beta_j$, $\|K_1(t)\| \le \gamma_1$, $\|J_jK_j(t)\| \le \varepsilon_j$, where $\alpha_1, \beta_j, \gamma_1$ and ε_j are known positive constants $(j = 2, 3, 4\cdots, N)$.

Remark 3. If the scaling matrix $J = I$, then the combination-combination projective synchronization problem will be reduced to combination-combination complete synchronization, where I is a $n\times n$ identity matrix.

Remark 4. If the scaling matrix $J = -I$, then the combination-combination projective synchronization problem will be reduced to combination-combination anticomplete synchronization.

Remark 5. If the scaling matrix $J = 0$, the combination-combination projective synchronization will be turned into a chaos control problem.

Remark 6. Definition 1 shows that the combination of drive systems and the combination of response systems can be extended to three or more chaotic systems.

Remark 7. There are many different scaling factors in a scaling matrix J, so all the dynamical states of the combination-combination projective synchronization can be adjusted of the different states independently.

As we all known, the main idea of synchronization control is to change the combination synchronization of chaotic systems problem into the combination synchronization error stability problem of the chaotic system.

Based on systems (1)-(4), the following conclusion can be drawn:

$$\begin{aligned}\dot{e}_{j-1}(t) &= \dot{x}_1(t) + \dot{y}_1(t) - J_j\dot{x}_j(t) - J_j\dot{y}_j(t) = A_1x_1(t) \\ &\quad + f_1(x_1(t)) + D_1(t) + B_1y_1(t) + g_1(y_1(t)) \\ &\quad + K_1(t) - J_j\left[A_jx_j(t) + f_j\left(x_j(t)\right) + D_j(t)\right. \\ &\quad + u_{j-1}(t) + B_jy_j(t) + g_j\left(y_j(t)\right) + K_j(t) \\ &\quad \left.+ w_{j-1}(t)\right] \\ &= A_1e_{j-1}(t) + B_1e_{j-1}(t) + \left(A_1J_j - J_jA_j\right)x_j(t) \\ &\quad + \left(B_1J_j - J_jB_j\right)y_j(t) + f_1(x_1(t)) + g_1(y_1(t)) \\ &\quad - J_jf_j\left(x_j(t)\right) - J_jg_j\left(y_j(t)\right) + D_1(t) + K_1(t) \\ &\quad - J_jD_j(t) - J_jK_j(t) - J_j\left[u_{j-1}(t) + w_{j-1}(t)\right],\end{aligned} \quad (7)$$

where $v_{j-1}(t) = u_{j-1}(t) + w_{j-1}(t)$ is the combination control controller ($j = 2, 3, \ldots, N$).

3. The Design of Sliding Mode Controller

In this section, combination-combination projective synchronization will be discussed and implemented for multiple different complex chaotic systems with unknown disturbances via sliding mode controller. Here, we take the following methods to achieve synchronization among multiple chaos systems. First, we should define a nonsingular terminal sliding surface $s_{j-1}(t)$ and, second, determine the control law to guarantee the existence of the sliding motion. The appropriate sliding mode surface is defined as follows:

$$s_{j-1}(t) = \lambda_{j-1}e_{j-1}(t), \quad (8)$$

where $s_{j-1}(t) = [s_{j-1,1}(t), s_{j-1,2}(t), \ldots, s_{j-1,n}(t)]^T$ is a vector and $\lambda_{j-1} = [\lambda_{j-1,1}(t), \lambda_{j-1,2}(t), \ldots, \lambda_{j-1,n}(t)]$ is a constant vector that need to be given ($j = 2, 3, \ldots, N$). Also, the reaching law is selected as

$$\dot{s}_{j-1}(t) = -q_{j-1}\operatorname{sgn}\left(s_{j-1}(t)\right) - r_{j-1}s_{j-1}(t), \quad (9)$$

where sgn$(\cdot)$ denotes for the signum function, $\operatorname{sgn}(s_{j-1}(t)) = |s_{j-1}(t)|/s_{j-1}(t)$. $q_{j-1} > 0$ and $r_{j-1} > 0$ are switching gains ($j = 2, 3, \ldots, N$).

The sliding motion exists with the error system trajectories moving on sliding surface and staying on it forever, if and only if

$$\begin{aligned}s_{j-1}(t) &= \lambda_{j-1}e_{j-1}(t) = 0, \\ \dot{s}_{j-1}(t) &= -q_{j-1}\operatorname{sgn}\left(s_{j-1}(t)\right) - r_{j-1}s_{j-1}(t) = 0.\end{aligned} \quad (10)$$

After choosing the corresponding sliding surface, we design the control law to drive the error system trajectories to go onto the sliding surface. Therefore, to ensure the existence of the sliding motion, the corresponding combination controller $v_{j-1}(t) = u_{j-1}(t) + w_{j-1}(t)$ is designed to ensure the existence of the sliding motion. The combination controller can be proposed as follows:

$$\begin{aligned}v_{j-1}(t) &= u_{j-1}(t) + w_{j-1}(t) \\ &= J_j^{-1}\left[\left(A_1J_j - J_jA_j\right)x_j(t) + \left(B_1J_j - J_jB_j\right)y_j(t)\right. \\ &\quad + f_1(x_1(t)) + g_1(y_1(t)) - J_jf_j\left(x_j(t)\right) \\ &\quad \left.- J_jg_j\left(y_j(t)\right)\right] - J_j^{-1}\rho_{j-1}\theta_{j-1}(t),\end{aligned} \quad (11)$$

where $\rho_{j-1} = [\rho_{j-1,1}, \rho_{j-1,2}, \ldots, \rho_{j-1,n}]^T$ is the constant gain that needs to be given ($j = 2, 3, \ldots, N$).

$$\theta_{j-1}(t) = \begin{cases}\theta_{j-1}(t)^{+} & s_{j-1} \geq 0 \\ \theta_{j-1}(t)^{-} & s_{j-1} < 0,\end{cases} \quad (12)$$

where $\theta_{j-1}(t)^{+}$ and $\theta_{j-1}(t)^{-}$ are right-hand limit and left-hand limit of the $\theta_{j-1}(t)$, respectively.

Substituting $v_{j-1}(t) = u_{j-1}(t) + w_{j-1}(t)$ from (11) into the above equation (8), the error system (7) can be further given as

$$\begin{aligned}\dot{e}_{j-1}(t) &= \dot{x}_1(t) + \dot{y}_1(t) - J_j\dot{x}_j(t) - J_j\dot{y}_j(t) \\ &= \left[A_1e_{j-1}(t) + B_1e_{j-1}(t) + D_1(t) + K_1(t)\right. \\ &\quad \left.- J_j\left(D_j(t) + K_j(t)\right) + \rho_{j-1}\theta_{j-1}(t)\right].\end{aligned} \quad (13)$$

So according to formulas (8), (9), and (13), we have the following result:

$$\begin{aligned}\theta_{j-1}(t) &= -\rho_{j-1}^{-1}\left[A_1e_{j-1}(t) + B_1e_{j-1}(t) + D_1(t)\right. \\ &\quad \left.+ K_1(t) - J_j\left(D_j(t) + K_j(t)\right) - \dot{e}_{j-1}(t)\right] \\ &= -\rho_{j-1}^{-1}\left[A_1e_{j-1}(t) + B_1e_{j-1}(t) + D_1(t) + K_1(t)\right. \\ &\quad \left.- J_j\left(D_j(t) + K_j(t)\right) - \lambda_{j-1}^{-1}\dot{s}_{j-1}(t)\right] \\ &= -\lambda_{j-1}^{-1}\rho_{j-1}^{-1}\left[\lambda_{j-1}A_1e_{j-1}(t) + \lambda_{j-1}B_1e_{j-1}(t)\right. \\ &\quad + \lambda_{j-1}D_1(t) + \lambda_{j-1}K_1(t) \\ &\quad - \lambda_{j-1}J_j\left(D_j(t) + K_j(t)\right) + q_{j-1}\operatorname{sgn}\left(s_{j-1}(t)\right) \\ &\quad \left.+ r_{j-1}s_{j-1}(t)\right].\end{aligned} \quad (14)$$

Obviously, the external disturbances are generally uncertain, so the control law $\theta_{j-1}(t)$ can be simply designed in the following form:

$$\theta_{j-1}(t) = -\lambda_{j-1}^{-1}\rho_{j-1}^{-1}\left[\lambda_{j-1}A_1 e_{j-1}(t) + \lambda_{j-1}B_1 e_{j-1}(t) + q_{j-1}\operatorname{sgn}\left(s_{j-1}(t)\right) + r_{j-1}s_{j-1}(t)\right]. \tag{15}$$

The proposed combination controller in (11) and the control law in (15) will guarantee the occurrence of the sliding motion, which is proved in the following Theorem 8.

Theorem 8. *Consider the error system (5) is controlled with the combination controller in (11) and the combination law in (15). If the following condition (16) is satisfied, then the errors of the system (5) will go toward the sliding surface and will reach the sliding surface, which means that the combination-combination projective synchronization of multiple chaotic systems is realized.*

$$\left\|\lambda_{j-1}\right\|\left(\alpha_1 + \gamma_1 + \beta_{j-1} + \varepsilon_{j-1}\right) - q_{j-1} < 0. \quad (j = 2, 3, \ldots, N). \tag{16}$$

Proof. Consider a positive definite function as a Lyapunov function candidate,

$$V_{j-1} = \frac{1}{2}\sum_{j=2}^{n}\left[s_{j-1}^{2}(t)\right], \quad (j = 2, 3, \ldots, N). \tag{17}$$

Its derivative with respect to time t along the trajectory is

$$\dot{V}_{j-1} = \sum_{j=2}^{n}\left[s_{j-1}(t)\,\dot{s}_{j-1}(t)\right]. \tag{18}$$

According to formulas (7) and (8), the result can be obtained

$$\begin{aligned}
\dot{V}_{j-1} &= \sum_{j=2}^{n}\left[s_{j-1}(t)\,\lambda_{j-1}\dot{e}_{j-1}(t)\right] \\
&= \lambda_{j-1}s_{j-1}(t)\left[\dot{x}_1(t) + \dot{y}_1(t) - J_j\dot{x}_j(t) - J_j\dot{y}_j(t)\right] \\
&= \lambda_{j-1}s_{j-1}(t)\left\{A_1 e_1 + B_1 e_1 + \left(A_1 J_j - J_j A_j\right)x_j(t)\right. \\
&\quad + \left(B_1 J_j - J_j B_j\right)y_j(t) + f_1\left(x_1(t)\right) + g_1\left(y_1(t)\right) \\
&\quad - J_j f_j\left(x_j(t)\right) - J_j g_j\left(y_j(t)\right) + D_1(t) + K_1(t) \\
&\quad \left. - J_j\left[D_j(t) + K_j(t) + u_{j-1}(t) + w_{j-1}(t)\right]\right\}.
\end{aligned} \tag{19}$$

Inserting $\dot{e}_{j-1}(t)$ and $\theta_{j-1}(t)$ from (13) and (15) into (19), one obtains

$$\begin{aligned}
\dot{V}_{j-1} &= \sum_{j=2}^{n} s_{j-1}(t)\,\lambda_{j-1}\dot{e}_{j-1}(t) \\
&= \lambda_{j-1}s_{j-1}(t)\left\{A_1 e_{j-1}(t) + B_1 e_{j-1}(t) + D_1(t)\right. \\
&\quad + K_1(t) - J_j\left[D_j(t) + K_j(t)\right] \\
&\quad - \rho_{j-1}\lambda_{j-1}^{-1}\rho_{j-1}^{-1}\left[\lambda_{j-1}A_1 e_{j-1}(t) + \lambda_{j-1}B_1 e_{j-1}(t)\right. \\
&\quad \left.\left. + q_{j-1}\operatorname{sgn}\left(s_{j-1}(t)\right) + \gamma_{j-1}s_{j-1}(t)\right]\right\} \\
&= \lambda_{j-1}s_{j-1}(t)\left\{D_1(t) + K_1(t) - J_j\left[D_j(t) + K_j(t)\right]\right\} \\
&\quad - s_{j-1}(t)\,q_{j-1}\operatorname{sgn}\left(s_{j-1}(t)\right) - \gamma_{j-1}s_{j-1}^{2}(t).
\end{aligned} \tag{20}$$

By the fact that $\operatorname{sgn}(s_{j-1}(t)) = |s_{j-1}(t)|/s_{j-1}(t)$, we gain

$$\begin{aligned}
&\dot{V}_{j-1} \\
&\quad = \lambda_{j-1}s_{j-1}(t)\left\{D_1(t) + K_1(t) - J_j\left[D_j(t) + K_j(t)\right]\right\} \\
&\quad - \left|s_{j-1}(t)\right| q_{j-1} - \gamma_{j-1}s_{j-1}^{2}(t).
\end{aligned} \tag{21}$$

According to Assumption 2, one obtains

$$\begin{aligned}
\dot{V}_{j-1} &= \sum_{j=2}^{n} s_{j-1}(t)\,\lambda_{j-1}\dot{e}_{j-1}(t) = \lambda_{j-1}s_{j-1}(t)\left\{D_1(t)\right. \\
&\quad \left. + K_1(t) - J_j\left[D_j(t) + K_j(t)\right]\right\} - \left|s_{j-1}(t)\right| q_{j-1} \\
&\quad - \gamma_{j-1}s_{j-1}^{2}(t) \le \left\|\lambda_{j-1}\right\|\left\|s_{j-1}(t)\right\|\left[\left\|D_1(t)\right\|\right. \\
&\quad \left. + \left\|K_1(t)\right\| + \left\|J_j\left(D_j(t)\right\| + \left\|J_j K_j(t)\right)\right\|\right] - \left|s_{j-1}(t)\right| \\
&\quad \cdot q_{j-1} - \gamma_{j-1}s_{j-1}^{2}(t) \le \left[\left\|\lambda_{j-1}\right\|\left(\alpha_1 + \gamma_1 + \beta_j + \varepsilon_j\right)\right. \\
&\quad \left. - q_{j-1}\right]\left\|s_{j-1}(t)\right\| - \gamma_{j-1}s_{j-1}^{2}(t).
\end{aligned} \tag{22}$$

In light of condition (16), we can easily obtain

$$\begin{aligned}
\dot{V}_{j-1} &\le \left[\left\|\lambda_{j-1}\right\|\left(\alpha_1 + \gamma_1 + \beta_j + \varepsilon_j\right) - q_{j-1}\right]\left\|s_{j-1}(t)\right\| \\
&\quad - \gamma_{j-1}s_{j-1}^{2}(t) < 0.
\end{aligned} \tag{23}$$

Hence, the proof is achieved completely. □

4. Numerical Simulations

In order to validate the efficiency and effectiveness of the proposed theory, the complex Lorenz system, the complex Chen system, the complex *Lü* system, and Rössler system with disturbance are proposed and corresponding controllers are designed.

The first drive system is the Lorenz system (24) and corresponding response system is the Chen system (25).

$$\begin{aligned} \dot{x}_{11} &= -10x_{11} + 10x_{12} + d_{11}, \\ \dot{x}_{12} &= 28x_{11} - x_{12} - x_{11}x_{13} + d_{12}, \\ \dot{x}_{13} &= x_{11}x_{12} - \frac{8}{3}x_{13} + d_{13}, \end{aligned} \tag{24}$$

and

$$\begin{aligned} \dot{x}_{21} &= -35x_{21} + 35x_{22} + d_{21} + u_{11}, \\ \dot{x}_{22} &= -7x_{21} + 28x_{22} - x_{21}x_{23} + d_{22} + u_{12}, \\ \dot{x}_{23} &= -3x_{23} + x_{21}x_{22} + d_{23} + u_{13}. \end{aligned} \tag{25}$$

The second drive system is the L*ü* system (26) and corresponding response system is the Rössler system (27).

$$\begin{aligned} \dot{y}_{11} &= -36y_{11} + 36y_{12} + k_{11}, \\ \dot{y}_{12} &= 20y_{12} - y_{11}y_{13} + k_{12}, \\ \dot{y}_{13} &= -3y_{13} + y_{11}y_{12} + k_{13}, \end{aligned} \tag{26}$$

and

$$\begin{aligned} \dot{y}_{21} &= -y_{22} - y_{23} + k_{21} + w_{11}, \\ \dot{y}_{22} &= -y_{21} + 0.2y_{22} + k_{22} + w_{12}, \\ \dot{y}_{23} &= -0.2 - 5.7y_{23} + y_{21}y_{23} + k_{23} + w_{13}. \end{aligned} \tag{27}$$

According to the given systems (24), (25), (26), and (27), the following conclusions can be obtained:

$$\begin{aligned}
A_1 &= \begin{bmatrix} 10 & 10 & 0 \\ 28 & -1 & 0 \\ 0 & 0 & -\frac{8}{3} \end{bmatrix}, \\
A_2 &= \begin{bmatrix} -35 & 35 & 0 \\ -7 & 28 & 0 \\ 0 & 0 & -3 \end{bmatrix}, \\
f_1(x_1) &= \begin{bmatrix} 0 \\ -x_{11}x_{13} \\ x_{11}x_{12} \end{bmatrix}, \\
f_2(x_2) &= \begin{bmatrix} 0 \\ -x_{21}x_{23} \\ x_{21}x_{22} \end{bmatrix}, \\
u_1 &= \begin{bmatrix} u_{11} \\ u_{12} \\ u_{13} \end{bmatrix}, \\
D_1 &= \begin{bmatrix} 0.1\cos(10t) \\ -0.2\cos(15t) \\ 0 \end{bmatrix}, \\
D_2 &= \begin{bmatrix} -0.1\cos(10t) \\ -0.1\cos(15t) \\ 0 \end{bmatrix}, \\
B_1 &= \begin{bmatrix} -36 & 36 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & -3 \end{bmatrix}, \\
B_2 &= \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0.28 & 0 \\ 0 & 0 & -5.7 \end{bmatrix}, \\
g_1(y_1) &= \begin{bmatrix} 0 \\ -y_{11}y_{13} \\ y_{11}y_{12} \end{bmatrix}, \\
g_2(y_2) &= \begin{bmatrix} 0 \\ 0 \\ -0.2 + y_{21}y_{23} \end{bmatrix}, \\
w_1 &= \begin{bmatrix} w_{11} \\ w_{12} \\ w_{13} \end{bmatrix}, \\
K_1 &= \begin{bmatrix} -0.2\cos(10t) \\ 0.1\cos(20t) \\ 0 \end{bmatrix}, \\
K_2 &= \begin{bmatrix} -0.2\cos(10t) \\ -0.1\cos(20t) \\ 0 \end{bmatrix}.
\end{aligned} \tag{28}$$

Assuming $J_2 = \mathrm{diag}\{1, -1, 2\}$, so the system dynamic errors can be obtained

$$\begin{aligned}
\dot{e}_{11} &= -10x_{11} + 10x_{12} + d_{11} - 36y_{11} + 36y_{12} + k_{11} \\
&\quad + 35x_{21} - 35x_{22} - d_{21} - u_{11} + y_{22} + y_{23} \\
&\quad - k_{21} - w_{11}, \\
\dot{e}_{12} &= 28x_{11} - x_{12} - x_{11}x_{13} + d_{12} + 20y_{12} - y_{11}y_{13} \\
&\quad + k_{12} - 7x_{21} + 28x_{22} - x_{21}x_{23} + d_{22} + u_{12} \\
&\quad + y_{21} + 0.2y_{22} + k_{22} + w_{12},
\end{aligned}$$

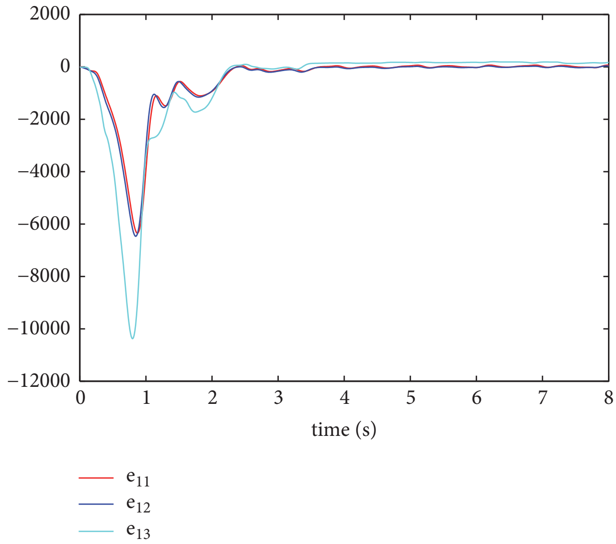

FIGURE 1: Synchronization errors e_{11}, e_{12}, e_{13}.

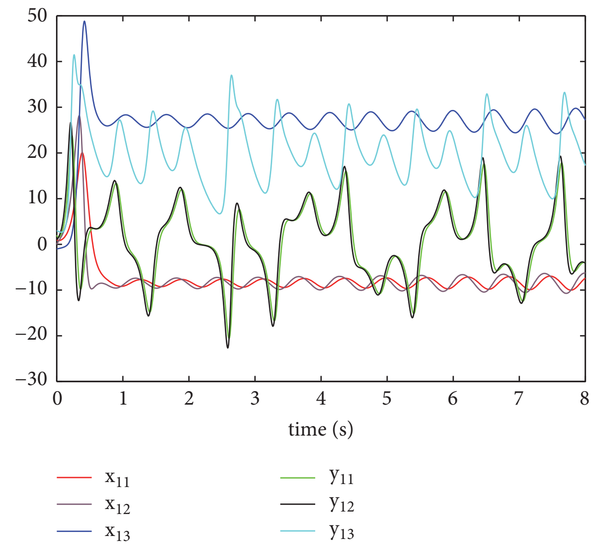

FIGURE 2: Drive state trajectories $x_{11}, x_{12}, x_{13}, y_{11}, y_{12}, y_{13}$.

$$\begin{aligned}\dot{e}_{13} = {} & x_{11}x_{12} - \frac{8}{3}x_{13} + d_{13} - 3y_{13} + y_{11}y_{12} + k_{13} \\ & + 6x_{23} - 2x_{21}x_{22} - 2d_{23} - 2u_{13} - 0.4 \\ & + 11.4y_{23} - 2y_{21}y_{23} - 2k_{23} - 2w_{13},\end{aligned} \tag{29}$$

Then, it is assumed that $\lambda_1 = [0, 1, 1]$ and $\rho_1 = [0, 1, 0]^T$, $\gamma_1 = 1$, and $q_1 = 2$.

Thus, the result can be obtained

$$\theta_1 = -28e_{11} - 20e_{12} - \frac{14}{3}e_{13} - 2\operatorname{sgn}(s_1), \tag{30}$$

that is,

$$\theta_1 = \begin{cases} 28e_{11} - 20e_{12} - \dfrac{14}{3}e_{13} - 2 & s_1 \geq 0 \\ 28e_{11} - 20e_{12} - \dfrac{14}{3}e_{13} + 2 & s_1 < 0. \end{cases} \tag{31}$$

In what follows, the numerical experiments are given to illustrate our results. In the simulation process, we assume the initial conditions of the drive chaotic systems and response chaotic systems are chosen as $(x_{11}(0), x_{12}(0), x_{13}(0)) = (1, 0, -1)$, $(x_{21}(0), x_{22}(0), x_{23}(0)) = (2, 3, 1)$, $(y_{11}(0), y_{12}(0), y_{13}(0)) = (2, 1, 3)$, $(y_{21}(0), y_{22}(0), y_{23}(0)) = (0, 2, 1)$. The state trajectories of the errors and corresponding variables are shown in Figures 1–7, respectively. According to the simulation parameters, it can be observed that the synchronization error e_1 converges to zero in Figure 1, which means that the combination-combination projective synchronization between two drive systems for Lorenz system (24) and Lü system (26) and two response systems for Chen system (25) and Rössler system (27) are realized via the sliding mode control. The drive state trajectories of systems (24) and (26) are shown in Figures 2 and 3. The response state trajectories of systems (25) and (27) are given in Figures 4 and 5. The state trajectories θ_1 are shown in Figure 6. The sliding mode surface motion trajectory s_1 is given in Figure 7.

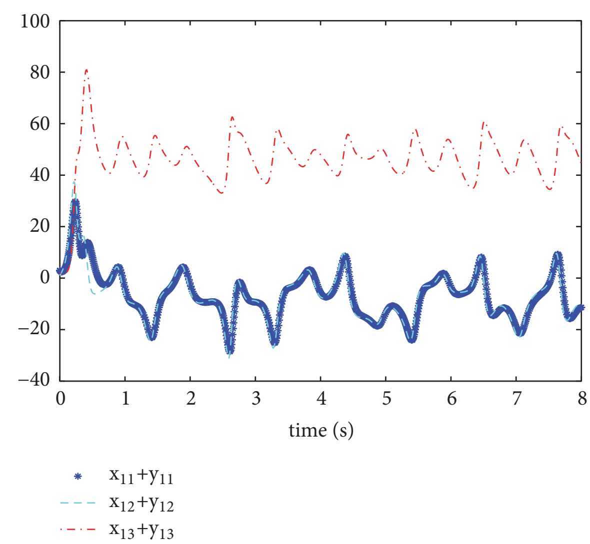

FIGURE 3: State trajectories $x_{11} + y_{11}, x_{12} + y_{12}, x_{13} + y_{13}$.

Remark 9. In the simulation, we should choose reasonable value according to the corresponding chaotic complex system to achieve desired result in the simple way.

Remark 10. The sliding control method is included in the discontinuous control methods as a special case. Hence, the sliding control method can be also applied in an array of coupled neural networks. The design of sliding control law is also a difficult point for the discontinuous control methods. The sliding mode control laws sometimes are too complex to realize in the real control. How to simplify the sliding mode control laws to reach the better effect is an interesting yet challenging problem during our future study.

Remark 11. It can be easily obtained that the drive systems and response systems can be the combination of nonidentical chaotic systems or identical chaotic systems.

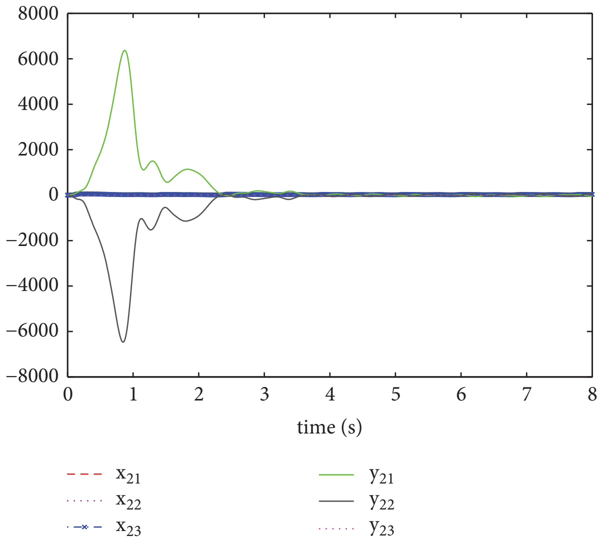

FIGURE 4: Response state trajectories $x_{21}, x_{22}, x_{23}, y_{21}, y_{22}, y_{23}$.

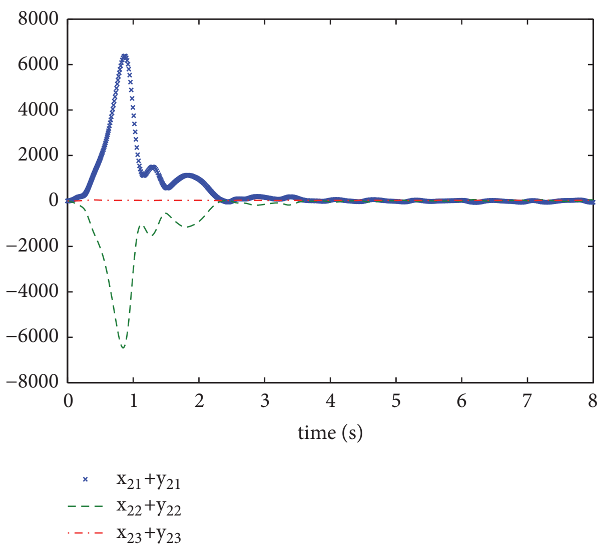

FIGURE 5: Response state trajectories $x_{21} + y_{11}, x_{22} + y_{22}, x_{23} + y_{23}$.

5. Conclusions

In this study, a novel modified combination-combination projective synchronization between the combination of two chaotic systems as the drive system and the combination of multiple chaotic systems as the response system with unknown parameters and disturbances are proposed. Furthermore, combined to the adaptive laws, the adaptive combination controllers and sliding mode manifold have been designed, and its convergence has been gained analytically. Finally, the simulations analyses have been given and have shown that the proposed combination controller works well for synchronizing the combination of drive systems and the combination of response systems. The proposed method may be more advantageous and have more potential than the

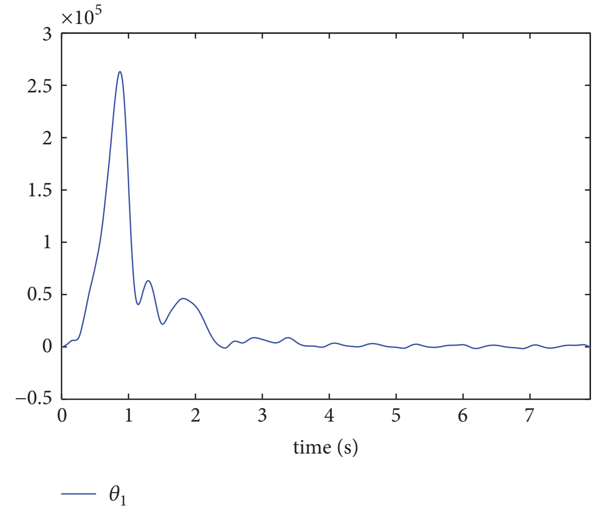

FIGURE 6: State trajectories θ_1.

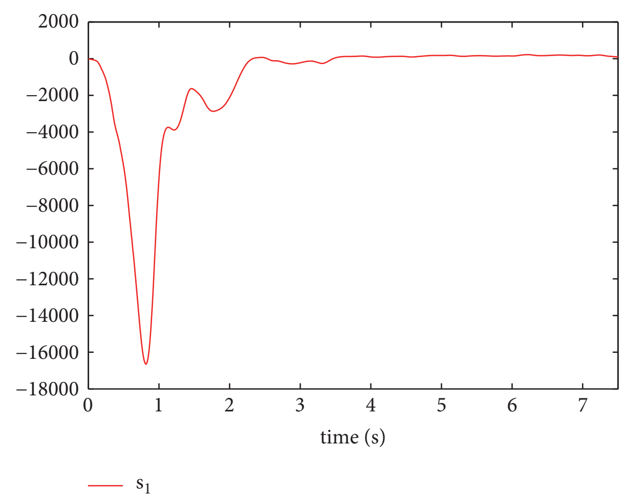

FIGURE 7: State trajectories s_1.

traditional control method to complete intelligent synchronization. How to realize combination-combination projective synchronization in actual practice is our next research topic.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

The work is supported by the State Key Program of National Natural Science of China (Grant no. 61632002), the National Key R and D Program of China for International S and T Cooperation Projects (no. 2017YFE 0103900), the National Natural Science of China (Grants nos. 61603348, 61775198, 61603347, 61572446, and 61472372), Science and Technology Innovation Talents of Henan Province (Grant no. 174200510012), Research Program of Henan Province (Grants nos. 172102210066, 17A120005, and 182102210160), and Youth Talent Lifting Project of Henan Province and the Science Foundation of for Doctorate Research of Zhengzhou University of Light Industry (Grant no. 2014BSJJ044).

References

[1] A. Akgul, S. Hussain, and I. Pehlivan, "A new three-dimensional chaotic system, its dynamical analysis and electronic circuit applications," *Optik - International Journal for Light and Electron Optics*, vol. 127, no. 18, pp. 7062–7071, 2016.

[2] Y. Scharf, "A chaotic outlook on biological systems," *Chaos, Solitons & Fractals*, vol. 95, pp. 42–47, 2017.

[3] S. Zhang and T. Gao, "A coding and substitution frame based on hyper-chaotic systems for secure communication," *Nonlinear Dynamics*, vol. 84, no. 2, pp. 833–849, 2016.

[4] S. Boccaletti, J. Kurths, G. Osipov, D. L. Valladares, and C. S. Zhou, "The synchronization of chaotic systems," *Physics Reports*, vol. 366, no. 1-2, pp. 1–101, 2002.

[5] J. Q. Lu and J. D. Cao, "Adaptive complete synchronization of two identical or different chaotic (hyperchaotic) systems with fully unknown parameters," *Chaos: An Interdisciplinary Journal of Nonlinear Science*, vol. 15, no. 4, Article ID 043901, 10 pages, 2005.

[6] Y. Xiao, W. Xu, S. Tang, and X. Li, "Adaptive complete synchronization of the noise-perturbed two bi-directionally coupled chaotic systems with time-delay and unknown parametric mismatch," *Applied Mathematics and Computation*, vol. 213, no. 2, pp. 538–547, 2009.

[7] Q. Song and J. Cao, "Synchronization and anti-synchronization for chaotic systems," *Chaos, Solitons & Fractals*, vol. 33, no. 3, pp. 929–939, 2007.

[8] S. Marchesini, Y.-C. Tu, and H.-T. Wu, "Alternating projection, ptychographic imaging and phase synchronization," *Applied and Computational Harmonic Analysis* , vol. 41, no. 3, pp. 815–851, 2016.

[9] S. Wang, X. Wang, and B. Han, "Complex generalized synchronization and parameter identification of nonidentical nonlinear complex systems," *PLoS ONE*, vol. 11, no. 3, article e0152099, 2016.

[10] R. G. Andrzejak, G. Ruzzene, and I. Malvestio, "Generalized synchronization between chimera states," *Chaos: An Interdisciplinary Journal of Nonlinear Science*, vol. 27, no. 5, 053114, 6 pages, 2017.

[11] Z.-M. Ge and C.-H. Yang, "Pragmatical generalized synchronization of chaotic systems with uncertain parameters by adaptive control," *Physica D: Nonlinear Phenomena*, vol. 231, no. 2, pp. 87–94, 2007.

[12] M. Ahmadlou and H. Adeli, "Visibility graph similarity: A new measure of generalized synchronization in coupled dynamic systems," *Physica D: Nonlinear Phenomena*, vol. 241, no. 4, pp. 326–332, 2012.

[13] H. Taghvafard and G. H. Erjaee, "Phase and anti-phase synchronization of fractional order chaotic systems via active control," *Communications in Nonlinear Science and Numerical Simulation*, vol. 16, no. 10, pp. 4079–4088, 2011.

[14] C. R. Mirasso, P. V. Carelli, T. Pereira, F. S. Matias, and M. Copelli, "Anticipated and zero-lag synchronization in motifs of delay-coupled systems," *Chaos: An Interdisciplinary Journal of Nonlinear Science*, vol. 27, no. 11, 114305, 8 pages, 2017.

[15] N. Novikov and B. Gutkin, "Robustness of persistent spiking to partial synchronization in a minimal model of synaptically driven self-sustained activity," *Physical Review E: Statistical, Nonlinear, and Soft Matter Physics*, vol. 94, no. 5, 2016.

[16] K.-Z. Li, M.-C. Zhao, and X.-C. Fu, "Projective synchronization of driving-response systems and its application to secure communication," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 56, no. 10, pp. 2280–2291, 2009.

[17] A. Bouzeriba, A. Boulkroune, and T. Bouden, "Projective synchronization of two different fractional-order chaotic systems via adaptive fuzzy control," *Neural Computing and Applications*, vol. 27, no. 5, pp. 1349–1360, 2016.

[18] P. Rao, Z. Wu, and M. Liu, "Adaptive projective synchronization of dynamical networks with distributed time delays," *Nonlinear Dynamics*, vol. 67, no. 3, pp. 1729–1736, 2012.

[19] J. Sun, Y. Shen, and G. Zhang, "Transmission projective synchronization of multi-systems with non-delayed and delayed coupling via impulsive control," *Chaos: An Interdisciplinary Journal of Nonlinear Science*, vol. 22, no. 4, Article ID 043107, 2012.

[20] A. E. Hramov and A. A. Koronovskii, "Time scale synchronization of chaotic oscillators," *Physica D: Nonlinear Phenomena*, vol. 206, no. 3-4, pp. 252–264, 2005.

[21] L. Runzi, W. Yinglan, and D. Shucheng, "Combination synchronization of three classic chaotic systems using active backstepping design," *Chaos: An Interdisciplinary Journal of Nonlinear Science*, vol. 21, no. 4, Article ID 043114, 2011.

[22] R. Luo and Y. Wang, "Finite-time stochastic combination synchronization of three different chaotic systems and its application in secure communication," *Chaos: An Interdisciplinary Journal of Nonlinear Science*, vol. 22, no. 2, Article ID 023109, 2012.

[23] J. Sun, Y. Shen, G. Zhang, C. Xu, and G. Cui, "Combination-combination synchronization among four identical or different chaotic systems," *Nonlinear Dynamics*, vol. 73, no. 3, pp. 1211–1222, 2013.

[24] J. Sun, Y. Shen, Q. Yin, and C. Xu, "Compound synchronization of four memristor chaotic oscillator systems and secure communication," *Chaos: An Interdisciplinary Journal of Nonlinear Science*, vol. 23, no. 1, Article ID 013140, 2013.

[25] G. H. Li, "Modified projective synchronization of chaotic system," *Chaos, Solitons & Fractals*, vol. 32, no. 5, pp. 1786–1790, 2007.

[26] X. Y. Wang, X. P. Zhang, and C. Ma, "Modified projective synchronization of fractional-order chaotic systems via active sliding mode control," *Nonlinear Dynamics*, vol. 69, no. 1-2, pp. 511–517, 2012.

[27] F. F. Zhang, S. T. Liu, and W. Y. Yu, "Stable adaptive fuzzy control of nonlinear systems," *Chin. Phys. B*, vol. 22, no. 12, pp. 141–151, 2013.

[28] Z. Wu and X. Fu, "Combination synchronization of three different order nonlinear systems using active backstepping design," *Nonlinear Dynamics*, vol. 73, no. 3, pp. 1863–1872, 2013.

[29] X. B. Zhou, M. R. Jiang, and Y. Q. Huang, "Combination synchronization of three identical or different nonlinear complex hyperchaotic systems," *Entropy*, vol. 15, no. 9, pp. 3746–3761, 2013.

[30] J. Sun, G. Cui, Y. Wang, and Y. Shen, "Combination complex synchronization of three chaotic complex systems," *Nonlinear Dynamics*, vol. 79, no. 2, pp. 953–965, 2015.

[31] J. Sun, Q. Yin, and Y. Shen, "Compound synchronization for four chaotic systems of integer order and fractional order," *EPL (Europhysics Letters)*, vol. 106, no. 4, Article ID 40005, 2014.

[32] B. Zhang and F. Q. Deng, "Double-compound synchronization of six memristor-based Lorenz systems," *Nonlinear Dynamics*, vol. 77, no. 4, pp. 1519–1530, 2014.

[33] H. R. Karimi and H. Gao, "New delay-dependent exponential H∞ synchronization for uncertain neural networks with mixed time delays," *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, vol. 40, no. 1, pp. 173–185, 2010.

[34] H. Tirandaz, H. Tavakoli, and M. Ahmadnia, "Modified projective synchronization of chaotic systems with noise disturbance, an active nonlinear control method," *International Journal of Electrical and Computer Engineering*, vol. 7, no. 6, pp. 3436–3445, 2017.

[35] Y. Yu and H. X. Li, "Adaptive hybrid projective synchronization of uncertain chaotic systems based on backstepping design," *Nonlinear Analysis: Real World Applications*, vol. 12, no. 1, pp. 388–393, 2011.

[36] S. Vaidyanathan, "Analysis, adaptive control and synchronization of a novel 4-D hyperchaotic hyperjerk system via backstepping control method," *Archives of Control Sciences*, vol. 26, no. 3, pp. 311–338, 2016.

[37] D. Xia, Y. Hu, and P. Ni, "A digitalized gyroscope system based on a modified adaptive control method," *Sensors*, vol. 16, no. 3, 2016.

[38] X. Liu, J. Cao, and G. Huang, "Complete periodic synchronization of delayed neural networks with discontinuous activations," *International Journal of Bifurcation and Chaos*, vol. 20, no. 7, pp. 2151–2164, 2010.

[39] H. Zhang, T. Ma, G.-B. Huang, and Z. Wang, "Robust global exponential synchronization of uncertain chaotic delayed neural networks via dual-stage impulsive control," *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, vol. 40, no. 3, pp. 831–844, 2010.

[40] D. Ghosh, I. Grosu, and S. K. Dana, "Design of coupling for synchronization in time-delayed systems," *Chaos: An Interdisciplinary Journal of Nonlinear Science*, vol. 22, no. 3, Article ID 033111, 8 pages, 2012.

[41] S. Vaidyanathan, S. Sampath, and A. T. Azar, "Global chaos synchronisation of identical chaotic systems via novel sliding mode control method and its application to Zhu system," *International Journal of Modelling, Identification and Control*, vol. 23, no. 1, pp. 92–100, 2015.

[42] S. Vaidyanathan, "Anti-synchronization of duffing double-well chaotic oscillators via integral sliding mode control," *International Journal of ChemTech Research*, vol. 9, no. 2, pp. 297–304, 2016.

[43] S. Singh, "Single input sliding mode control for hyperchaotic Lu system with parameter uncertainty," *International Journal of Dynamics and Control*, vol. 4, no. 4, pp. 504–514, 2016.

[44] L. Zhao and G.-H. Yang, "Adaptive sliding mode fault tolerant control for nonlinearly chaotic systems against DoS attack and network faults," *Journal of The Franklin Institute*, vol. 354, no. 15, pp. 6520–6535, 2017.